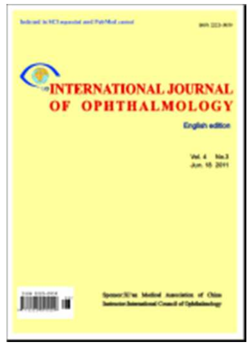

Human eye ocular components analysis for refractive state and refractive surgery

Journal:	*International Journal of Ophthalmology*
Manuscript ID	IJO-2016-0696.R1
Manuscript Type:	Original Article
Date Submitted by the Author:	n/a
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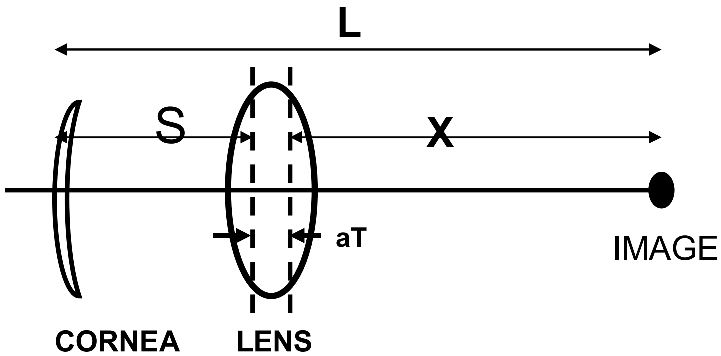
Keywords:	Gaussian optics, human eye ocular components, refractive errors, vision correction, LASIK, Crosslink
Speciality:	Optometry & Ophthalmology, Eye laser treatment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Human eye ocular components analysis for refractive state and refractive surgery

Running title: Human eye optics analysis

ABSTRACT

AIM: To analyze the clinical factors influencing the human vision corrections via the changing of ocular components of human eye in various applications; and to analyze refractive state via a new effective axial length.

METHODS: An effective eye model is introduced by the ocular components of human eye including refractive indexes, surface radius (r1, r2, R1, R2) and thickness (t,T) of the cornea and lens, the anterior chamber depth(S1) and the vitreous length (S2). Gaussian optics is used to calculate the change rate of refractive error per unit amount of ocular components of a human eye (the rate function M). A new criterion of myopia is presented via an effective axial length.

RESULTS: For typical corneal and lens power of 42 and 21.9 diopters, the rate function Mj (j=1 to 6) are calculated for a 1% change of r1, r2, R1, R2, t, T (in diopters) M1=+0.485, M2=-0.063, M3=+0.053, M4=+0.091, M5=+0.012,and M6=-0.021 diopters. For 1.0 mm increase of S1 and S2, the rate functions are: M7=+1.35, and M8=-2.67 diopter/mm. These rate functions are used to analyze the clinical outcomes in various applications including laser in situ keratomileusis (LASIK) surgery, corneal cross linking (CXL) procedure, femtosecond laser surgery and scleral ablation for accommodation.

CONCLUSION: Using Gaussian optics, analytic formulas are presented for the change of refractive power due to various ocular parameter changes. These formulas provide useful clinical guidance in refractive surgery and other related procedures.

Keywords: *Gaussian optics, human eye ocular components, refractive errors, vision correction LASIK CXL*

INTRODUCTION

A complete optical description of a human eye should include its 12 ocular parameters including 4 refractive indexes, 4 surface radius and 2 thickness (for cornea and lens), the anterior chamber depth and the vitreous length (or axial length). Gaussian optics [1,2] has been used for the calculations of intraocular lens (IOL) power, accommodation amplitude in IOL and human natural lens and the refractive state of human eyes[3,4]. Conventional refractive state is defined solely by the axial length (L) which could not apply to all eyes, although it is true for averaged eyes. Base on an effective eye model, a new standard for refractive state will be presented based on a relative axial length of (L-L*), rather than its absolute axial length (L), where L* is the effective axial length of the emmetropic state. The roles of ocular components on the refractive power have been reported only partially [2,3]. Derivation of the rate function (M) defined by the change rate of refractive error per unit amount of ocular components will be presented else where. This study will focus upon their clinical applications including laser in situ keratomileusis (LASIK) surgery, corneal cross linking (CXL) procedure, femtosecond laser surgery and laser scleral ablation for accommodation.

MATERIALS AND METHODS

The Effective Eye Model

By Gaussian optics theory (or paraxial ray approximation along the axial axis), the refractive error (De) is given by [1,3]

$$De = 1000\,[n1/(L-L2) - n1/F], \qquad (1)$$

where n1 is the refractive index of the aqueous humor, L is the axial length, L2 is position of the system second principal plane and F is the system effective focal length (EFL). The system total power is given by D=1000n1/F (D in diopter, F in mm) which is determined by the corneal (D1) and lens power (D2) as follows [3]

$$D = D1 + D2 - S(D1D2)/(1000n1), \qquad (2.a)$$

$$D1 = 1000\,[(n3-1)/r1 - (n3-n1)/r2] + bt, \qquad (2.b)$$

$$D2 = 1000\,[(n4-n1)/R1 + (n4-n2)/R2] - aT, \qquad (2.c)$$

where nj (j=1, 2, 3, 4) are the refractive index for the aqueous, vitreous, cornea and lens, respectively. The anterior and posterior radius of curvatures (in the unit of mm) of the cornea and lens are given by (r1, r2) and (R1, R2), respectively, where the only concave surface R2 is taken as its absolute value in this study. Finally, S is the effective anterior chamber depth, related to the anterior chamber depth (ACD), S1, by S=S1+P11+0.05 (in mm), where P11 is the distance between the lens anterior surface and its first principal plane, and 0.05 mm is a correction amount to include the effect of corneal thickness (assumed to be 0.55 mm) [2,3]. The thickness terms in Eq.(2.b) and (2.c) are given by b=11.3/(r1r2), a=4.97/(R1R2) for refractive indexes of n1 = n2 = 1.336, n3 = 1.377 and n4 = 1.42; and t and T are the thickness of the cornea and lens, respectively.

As shown in Fig. 1, using L-L2=X+ SF/f, with X=L-S-aT+0.05, and aT and 0.05 are the correction factors for the lens and cornea thickness, Eq.(1) may be rewritten in an effective eye model equation [3]

$$De = Z^2\,[1336/X - D1/Z - D2] \qquad (3.a)$$

$$Z = 1 - S/f \qquad (3.b)$$

where f (in mm) is the EFL of the cornea given by f=1336/D1, and the nonlinear term k is about 0.003 calculated from the second-order approximation of SF/(1336f). The nonlinear term may also be derived from the IOL power formula [5]. We note that in Eq. (3), X, Z, S and f are in the unit of mm; D1, D2 and De are in the unit of diopter; and the 1336 is from 1000x1.366 in our converted units.

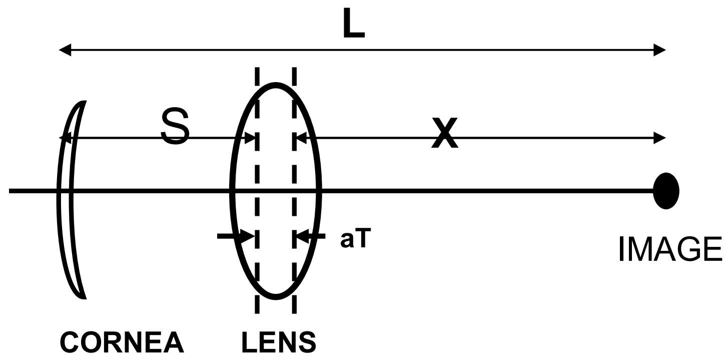

Fig. 1. An effective eye model [3] defined by the power of the cornea and lens. Also shown are the parameters of S and X which is related to the axial length by L=S+X+aT - 0.05 (mm).

A New Standard of Refractive State[3]

The emmetropic state ("E-state", when De=0) can be described by a simple formula reduced from Eq.(3.a) when 1336/X = D1/Z + D2, or as shown by Fig, 1, when the effective axial length at E-state (L*) is given by [3] L* = X+S+aT-0.05, which also define the refractive states for hyperopia De>0 (L<L*), and myopia De<0 (L>L*). We may also easily see that at emmetropia De=0, or when L=L*. Therefore, a new standard for E-state is governed by the relative axial length of (L-L*), rather than its absolute axial length (L). A large L* may be due to flat cornea or lens (i.e. small D1 or D2) or deep anterior chamber depth (S), or thick lens (T). The commonly accepted concept of long axial length resulting myopia is only true under statistical "mean". The refractive state of a specific subject shall be defined by our new criterion as described above. For example, a subject with L=26 mm will have about 2.7 diopter myopia when L*=25 mm, whereas it becomes about 1.4 diopter of hyperopia, when L*=27 mm. The above new standard for E-state was first introduced by Lin in 2006 [3]. Using the referenced parameter set of (f1, f2, So, T, L*)=(31, 60, 3.3, 4.0, 24) mm, an ocular system deviating from this referenced-set, its emmetropic state is governed by [3]

$$L^*=24.0+0.36(43.1-D1)+0.23(22.3-D2)+0.5(So-3.3)+0.35(T-4.0) \quad (4)$$

The Rate Functions

To find the change of refractive error (De) due to the change of Qj, we further define Qj=(r1, r2, R1, R2, t, T, S1, S2) with j=(1 to 8), respectively. The ACD (S1) and vitreous length (S2) are related to the axial length by L=S1+S2+T. The derivative of the refractive error (De) with respect to these ocular parameter change (Qj) given by Mj=dDe/dQj, defines the rate function, or the change of De per unit amount change of Qj, where the standard notation "d" for "derivative" is used in this study.

In general, under the second-order approximation including the contributions from both n1/(L-L2) and (n1/F) in Eq.(1), one shall rigorously calculate the derivative dDe=Mj(dQj) based on Eq.(1). The complexity of this method is due to the nonlinear dependence of L2 on the ocular parameters.

Using Eq. (2) and (3) analytic formulas for the rate function for the surface curvatures and thickness of the cornea and lens may be derived (to be presented else where) by Mj=dDe/dQj, with Qj (j= 1 to 4, for r1, r2, R1 and R2, respectively), and Q5=t, Q6=T as follows.

$$M1 = +378/r1^2, \quad (5.a)$$
$$M2 = -41/r2^2, \quad (5.b)$$
$$M3 = +82.75\ C_2/R1^2, \quad (5.c)$$
$$M4 = +82.75\ C_2/R2^2, \quad (5.d)$$
$$M5 = 11.3 / (r1r2), \quad (5.e)$$
$$M6 = +4.97\ C_2/(R1R2). \quad (5.f)$$

where we had used the refractive indexes nj=(1.336, 1.336, 1.3371, 1.42) for the aqueous, vitreous, cornea and lens, respectively, and a lens conversion function $C_2 = (dDe/dD2)=Z^2$.

The rate function for S1 and S2 , defined by M7=dDe/dS1 and M8=dDe/dS2, were previously derived and given by [4-6]

$$M7= 1336\ (1/F^2 - 1/f^2), \quad (6.a)$$
$$M8= - 1336/F^2, \quad (6.b)$$

where f and F (both in mm) are the corneal and system EFL given by f=1336/D1 and F=1336/D.

For Mj=dDe/dQj, with Q(j=9,10,11,2) for nj (j=1,2,3,4), respectively, we derive (to be presented else where)

$$M9= 1000\ (1/r2 - C_2/R1) \quad (7.a)$$
$$M10 = - 1000C_2/R2, \quad (7.b)$$
$$M11= - 1000\ (1/r2 - 1/r1), \quad (7.c)$$
$$M12 = -1000\ C_2\ (1/R1 + 1/R2) \quad (7.d)$$

RESULTS

The rate functions

By using a set of typical ocular parameters[2]: refractive indexes nj (i=1 to 4) =(1.336, 1.336, 1.3771, 1.42), (r1, r2)=(7.8, 6.5) mm, (R1, R2)=(10.2, 6.0) mm, thickness (t, T)=(0.55, 4.0) mm and S=6.0, S1=3.5 and S2=16.0 mm, or an axial length of L=3.5 + 16 + 4 = 23.5 mm, the corneal and lens power are calculated D1=42 diopter, D2=21.9 diopter and total power, from Eq.(2.a), D=D1+0.811D2=59.8 diopter, The rate function Mj (j=1 to 6) are calculated for a 1% change of r1, r2, R1, R2, t, T (in diopters) M1=+0.485, M2=-0.063, M3=+0.053, M4=+0.091, M5=+0.012,and M6=-0.021 diopters.

For 1.0 mm increase of S1 and S2, the rate functions are: M7=+1.35, and M8=-2.67 diopter/mm. Furthermore, for each 1.0 diopter increase of corneal and lens power, the rate functions are 1.0 and 0.66 diopter, respectively, for a typical value of effective ACD, S=6.0 mm and corneal power of 43 diopters. We shall note that the above values of Mj depend on the choices of the ocular parameters and may vary 10% - 15% from the typical values chosen. Our calculated data are consistent with that of Ref. 2.

Effects of Cornea and Lens Curvatures

The increase of radius of curvature of the cornea and lens (r1, r2, R1, R2) all result in hyperopic shift, except the change of the posterior surface of the lens (R2) having a myopia shift, since it is the only concave surface and all other three surfaces (r2, R1, R2) are convex surfaces. Furthermore, the effect due to anterior corneal surface change is the dominant one, where M1 is about 8 times of M2 and M3, and 5 times of M4, as shown by Eq. (5). This may be easily realized from Eq.(2.b) that (n3-1) is much higher than the other terms, such as (n3-n1) and (n4-n1). Therefore reshaping of lens surface is much less efficient than that of cornea. We will discuss more later in femtosecond laser procedure.

Effects of S1 and S2

The increase of S1 results in a hyperopia shift (HS), whereas S2 results in a myopia shift (MS) , where M8 is about two times of M7 which has two competing terms as shown by Eq.(6). The rather high change rate M8=-2.67 (D/mm) has significant impact on the onset of emmetropization and myopia which are governed by the correlation among the growth of axial length (L=S1+S2+T) and the power decrease of the cornea and lens when an eye grows [3]. The change rate M7 having a lower value than M8 can be analyzed as follows.

The competing between the MS (due to the increase of ACD, S1) and the HS (due to the associate decrease of S2 for a fixed axial length L=S1+S2+T) results in a net hyperopic-shift, because the hyperopic component is always the dominant one, since the corneal power (D1) is always less than the total system power (D) or F<f in Eq.(3.a). This new finding based on the analytic formula of Eq.(5) has not been explored before.

The hyperopic shift due to the increase of S1 is equivalent to a myopic-shift when S1 decreases, or a forward movement of the lens. This feature is important for presbyopia accommodation which is contributed by two components: the lens curvature decrease and the lens forward movement [3,4]. The lens forward movement is also the main feature in an accommodative IOL and our formulas, Eq. (6) for M7 and M8 provide the amount of accommodation.

Effects of Refractive Index

The refractive error change (dDe) is extremely sensitive to the refractive indexes, about 0.3 to 2.5 diopters per 1% change. The increase of n1 and n4 result in a myopic-shift (MS), whereas the increase of n2 and n3 result in a hyperopic-shift (HS). These opposite behavior may be readily observed from Eq.(7). One may also find from Eq.(8.a) the reason why m2 is larger than m1. This is due to the minus term C_2/R1 in Eq.(7.a) and r2 < R1, in general, which results in an MS. The HS of m2 is given by Eq.(8.b), where R2 is defined as the absolute value of lens posterior radius in this study. Eq.(7.c) clearly shows that m3 has an MS due to the fact that r2 is always smaller than r1, without exceptions in all human eyes. Finally, the increase of lens refractive index (n4) always results in an MS, or becomes more power as expected from Eq.(5.d) and n4=1.42 is always larger than n1 and n2 in Eq.(2.c).

It should be emphasized that the new feature of m1, based on Eq.(7.a), is not obvious due to the contribution of the second term C_2/R1 involving a rather complex mathematics to derive the formula for C_2 which has been ignored in most textbook formulas[2]. Another interesting situation is when both n1 and n2 increase the same amount of 1% (the most likely case, since the aqueous and vitreous humor are circulated, the net effect will be dDe=-1.19 + 1.46 = +0.27, a hyperopic-shift only about 18% of dDe due to the change of n2 alone and shows a much less effect than that is due to the lens index change M12=2.47.

DISCUSSIONS

Clinical Applications

We will present various applications related to the formulas presented in this paper, including: laser in situ keratomileusis (LASIK) surgery, corneal cross linking (CXL) procedure, femtosecond laser surgery and accommodative IOL. Greater details are described as follows.

LASIK surgery [7]

A procedure called laser in situ keratomileusis (LASIK), where one diopter correction only requires an ablation depth about 8 to 11 microns of the corneal central thickness [6] or a corresponding change of r1 about 0.16 mm based on Eq.(5.a). It is important to know that the corneal power change is 100% converted to the system power or refractive error change, as demonstrated by our cornea conversion factor C_1. We should also note that the refractive error (De) defined on the corneal plan is the same as that of a contact lens. However, a conversion formula is needed when it is translated to a spectacle power Ds, given by De= Ds/ [1 - V Ds] , where V is a vertex distance about 12 mm.

The central ablation depth for a 3-zone myopic correction is given by [7]

$$H'(3\text{-zone}) = RH(\text{single-zone}), \qquad (8.a)$$

$$H(\text{single-zone}) = (DW^2/3)(1+C) \qquad (8.b)$$

where W is the diameter of the outer ablation zone having a typical value of 6.5 to 7.5 mm; C is a nonlinear correction term given by $C= 0.19\ (W/r1)^2$, r1 is the corneal anterior radius of curvature. For examples, for r1=7.8 mm, (or a K-reading of K=337.R1=43.2 D), C = (11.2, 13.2,16.5) % for W =(6.0, 6.5, 7.00 mm. The reduction factor R=(0.70 to 0.85) depending on the algorithms used. For example, comparing to a single zone with W=6.5 mm, a 3-zone depth will reduces to 71.6% (or R = 0.716) when an inner zone 5.5 mm and an outer zone 6.5 mm are used. Furthermore, in a LASIK system, the input pre-operative parameter of the treated eye must include the K values which affect the laser ablation depth via the nonlinear term of Eq. (8.b).

Age dependent lens power[8-10],

It was reported that the change in the refractive index gradient of the lens cortex has a substantial factor in the contribution to the onset and progress of presbyopia [8], where an age-dependent equation for an equivalent lens index neff=1.441 – 0.00039 x Age (in year) was proposed to explain the lens paradox [9]. Lens index decreases from 1.434 to 1.416 (about 1.25% decrease) between 20 and 65 years of age to compensate the more convex shape of aged-lens, given by R1=12.9 – 0.057xAge and R2=6.2-0.012 x Age [10], which would have caused a myopia rather than presbyopia, if neff would not be age-dependent. Above statements have been known, but only qualitatively. The formula Eq. (7.d) provides the quantitative argument that a hyperopia shift (HS) of 2.47 x 1.25% = 3.1 diopter is associated to this proposed index decrease of 1.2%. The commonly accepted estimation of dDe due to the change of lens index was based on a conversion factor (C_2) of 80% which ignored the contribution from the second principal plane, the first term of Eq.(1) in comparing to the new value of CF=(65% to 75%) in this study which includes both terms.

Accommodative IOL (AIOL) in aphakic eye [5,6, 11,12]

For patient after cataract, an AIOL may be implanted for vision correction to see both near and far. The accomondation formulas for M7 and M8 can be used to calculate the accomondation amplitude of the AIOL. Our calculations show the typical values of M7=+1.35, and M8=-2.67 diopter/mm. These formulas can also be used to calculate the power error of the piggy-back IOL due to mis-position. Our formulas based on the Gaussian optics are consistent with that of raytracing methods [11,12]

Femtosecond laser surgery

One may use a femtosecond laser to ablate or remove a small portion of the lens and change its curvature (R1), where each 1% reduction may cause a 0.05 to 0.06 diopter change, based on our formula for M3, see Eq. (5.c). This procedure is not as effective as that of corneal ablation (LASIK) given by M1 in Eq. (5.a). However, ablation of the lens has no thickness limitation like a cornea. Therefore one may ablate the lens to restore a 40% change of R1 resulting 2.0 to 2.4 diopter accommodation. The current femtosecond laser has a very low average power and therefore lens ablation could take a much longer time than a corneal surface ablation in LASIK.

Scleral ablation for presbyopia treatment [8]

Scleral laser ablation and band expansion have been used to increase the space of the ciliary-body and zonus such that accomondation is improved by two components [8]: the lens translation and the lens shaping which are given by, respectively, M7 and M3. For older and/or harder lens, the accommodation is mainly attributed by the lens translation (or S1 change), whereas lens shaping dominates the power change in young or soft lens. It was known that change of the rear surface of the lens is about one-third of the front surface during accommodation [12], our formulas Eq. (5.c) and (5.d) shows that the contribution from R2 is about the same as that of R1, because of R2 (6.0 mm) <R1(10.2 mm), and M4=2.9 M3, for the same change of curvature, dR1= dR2.

Cornea cross linking [13-20]

Depending on the ocular location of the corneal cross linking (CXL) procedure, the new applications of CXL include examples shown as follows:

(1) For CXL applied inside the corneal stroma, correction of low myopia is possible and may be measured by the K-value (or thickness) reduction after CXL; where 2% reduction of K-value may cause a 0.9 to 1.1 diopter myopic correction, based on the formula for M1, see Eq. (5.a), where K=337/r1. We shall note that the refractive power change based on M1 calculated by the K-value change may be underestimated, because the CXL could change both the front and back surface of the cornea resulted by the thickness reduction after the CXL. A more accurate calculation should include both M1 and M2 shown by Eq. (5).
(2) For CXL applied to the orbital scleral tissue, one may stop or reduce the abnormal axial length (L) growth rate in high myopic eyes; where each 1.0 mm increases of L may cause 2.2 to 2.8 diopter change, based on our formula for M8, see Eq. (6.b), assuming that the axial grow is dominated by S2.
(3) For CXL applied to the corneal stroma postoperatively for procedures such as conduction keratoplasty (CK), diode laser thermal keratoplasty (DTK), the postoperative regression due to unstable thermal shrinkage may be stabilized by CXL process. Eq. (5.a) for M1 may be used to estimate the amount of postoperative regression reduced by CXL.

CONCLUSION

Using Gaussian optics, we have presented analytic formulas for the change of refractive power due to various ocular parameter changes. These formulas provide useful clinical guidance in various applications including laser in situ keratomileusis (LASIK) surgery, corneal cross linking (CXL) procedure, femtosecond laser surgery and scleral ablation for accommodation. Accuracy of our formulas for human eyes would depend on individual ocular parameters, which were taken as their averaged values in our calculations. Moreover, we have assumed a simplified paraxial approximation eye mode (along the optical axis, z) which does not include the (x,y) off axis surface effects. Therefore the formulas developed in this article would only provide a general trends for clinical guidance, rather than accurate prediction for refractive surgeries in human eyes, in which a full 3-dimensiotinal model is required and only numerical simulation are available. Our intent of this article is to present comprehensive model with analytic formulas.

REFERENCES

1. Pedrotti LS , Pedrotti F, Optics and Vision. Liper Saddle River, USA, Prentice Hall; 1998.
2. Atchison DA, Smith G. Optics of the human eye. Woburn, USA, Butterworth Heinemann; 2000.
3. Lin JT. Analysis of refractive state ratios and the onset of myopia. Ophthal Physiol Opt. 2006;26: 97-105.
4. Lin JT, Jiang MS, Hong YL, et al. Analysis and applications of accommodative lenses for vision corrections. J Biomed Optics**.** 2010**;**16:018002-018002-5.
5. Garg A, Lin JT, editors. Mastering IOLs: Principles, Techniques and Innovations, India, Jaypee Brothers; 2007.
6. Lin, JT, "New formulas comparing accommodation in human lens and intraocular lens. J. Refract Surg. 2005; 21, 200-201.
7. Garg A, Lin JT, editors. Mastering the Advanced Surface Ablation Techniques, India, Jaypee Brothers; 2008.
8. Lin JT, Mallo O. Treatment of presbyopia by infrared laser radial sclerectomy. J. Refract Surg 2003: 19:465-467.
9. Rosem AM, Denham DB, Fernandez V et al, In vitro dimensions and curvatures of human lenses, Vision Res. 2006:46:1006-1009.
10. Garner LP, Yap MKH, Change in ocular dimensions and refraction with accommodation. Ophthal Physiol Opt. 1992, 17:12-17.
11. Nawa Y, Ueda T, Nakasuka et al. "Accommodation obtained per 1.0 mm forward movement of a posterior chamber IOL. J Cataract Refract Surg. 2003;29:2069-2072.
12. .Ho A, Manns F, Pham T et al. Predicting the performance of accommodating IOL using ray tracing. J Cataract Refract Surg. 2006; 32**:**129-136.
13. Hafezi F and Randleman JB. editors. Corneal Collagen Cross-linking, First ed. Thorofare (NJ): SLACK; 2013.
14. Sorkin N, Varssano D. Corneal collagen cross-linking: a systematic review. *Ophthalmologica*. 2014;232:10-27.
15. Lin JT. Analytic formulas for the clinical issues of a UV-light-activated corneal cross-linking device. J Biomed Eng Devic. 2016; 1:104. doi:10.4172/jbemd.1000104.
16. Lin JT, Cheng DC, Chang C, Yong Zhang. The new protocol and dynamic safety of UV-light activated corneal collagen cross-linking. Chin J Optom Ophthalmol Vis Sci 2015; 17:140-147.
17. Lin JT, Wang KC. Analytic formulas and numerical simulations for the dynamics of thick and non-uniform polymerization by a UV light, J Polymer Research, 2016; 23:53-59.
18. Spoerl E, Mrochen M, Sliney D, Trokel S, Seiler *T.* Safety of UVA-riboflavin cross-linking of the cornea*. Cornea.* 2007*;* 26*:* 385*–389.*
19. Lanchares E del Buey MA Cristóbal JA Lavilla L Calvo B. Biomechanical property analysis after corneal collagen cross-linking in relation to ultraviolet A irradiation time. Graefes Arch Clin Exp Ophthalmol . 2011; 249: 1223–1227.
20. Schumacher S, Mrochen M, Wernli J, Bueeler M, Seiler T. Optimization model for UV-riboflavin corneal cross-linking. *Invest Opthamol Vis Sci*. 2012;53:762-769.